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A Small Note about the Electric Field Radiated From a Large Open Ended Rectangular Waveguide

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1. Abstract

Formulate the performance of electric field inside the waveguide has been studied extensively, so that we work the expressions for the electric field radiated from a large open ended waveguide aperture for the propagating TE mode, to obtain the near field in the end of the guide. The reflection coefficient has been interpreted as the ratio of the reflected and incident electric field. Instead, we have used the reflection coefficient, considering the incidence angle, operation frequency and guide dimensions in order to calculate the electric field radiation pattern, plane *E* and plane *H*.

Key words: reflection coefficient, electric field, TE_{10} propagation mode.

2. Resumen (Una pequeña nota acerca del campo eléctrico radiado por una guía de onda rectangular larga abierta en el extremo)

Establecer el comportamiento del campo eléctrico dentro de una guía de onda ha sido bastante estudiado, así que hemos trabajado las expresiones para el campo eléctrico radiado por una guía de onda larga y abierta en un extremo para el modo de propagación TE, para obtener el campo cercano el extremo de la guía. El coeficiente de reflexión ha sido interpretado como la razón del campo eléctrico reflejado e incidente. En vez de esto, hemos usado el coeficiente de reflexión considerando el ángulo de incidencia, la frecuencia de operación y las dimensiones de la guía, con el objeto de determinar el patrón de radiación del campo eléctrico en el plano *E* y plano *H*.

Palabras clave: coeficientes de reflexión, campo eléctrico, modo de propagación TE_{10}.

3. Introduction

The radiation from a waveguide can be considered to arise from the current distribution on the inside walls of the guide, which is just the current distribution associated with the fields propagated in the interior of the guide. The guide allows propagation of only one mode, the dominant mode, the single mode field solutions, having fixed transverse distributions and limited variability in the axial direction, will be incapable of satisfying boundary conditions for the fields over the surface of a conducting or dielectric obstacle of arbitrary shape extending in three dimensions in the waveguide. Inside the guide, over a cross section inside the guide far from the open ended, the field is formed with the incident and reflected waves of the dominant mode, but in the aperture, the discontinuity in this point, additional higher mode fields exist, most of the higher mode terms above the dominant mode will have cutoff frequencies greater than the operating frequency of the waveguide and thus will not propagate away from the vicinity of the discontinuity which causes them to arise, producing radiation outside of the guide in the z direction, so that, the yz plane is the plane-*E*, and the xz plane is the *H*-plane of the system.

4. The electric field radiated

A. Analytical formulation

To obtain an exact solution for the electromagnetic fields in the neighborhood of an obstacle of irregular shape, in this

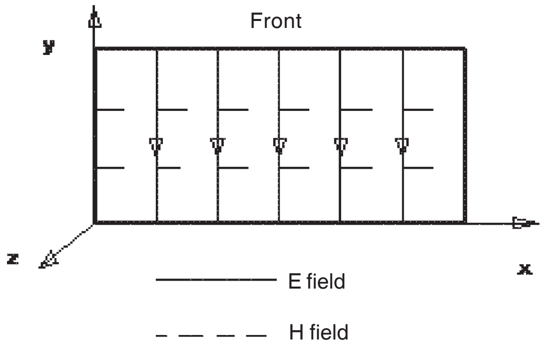


Fig. 1. Electric fields patterns for TE_{10} mode in a rectangular waveguide.

case, the free space, it is difficult. However, starting from the transverse fields the TE mode field equations in rectangular coordinates are well known [1]:

$$H_{tn} = \frac{j\beta}{k_c^2} \nabla H_{zn} \tag{1a}$$

$$E_{tn} = \frac{j\omega\mu}{k_c^2} \hat{z} \times \nabla H_{zn} \tag{1b}$$

$$\vec{E}_n \quad \vec{E}_{tn} \quad \hat{z} E_{zn} \tag{2a}$$

$$\vec{H}_n \quad \vec{H}_{tn} \quad \hat{z} H_{zn} \tag{2b}$$

$$\vec{E}_{tn} \quad \hat{x} E_{xn} \quad \hat{y} E_{yn} \tag{3a}$$

$$\vec{H}_{tn} \quad \hat{x} H_{xn} \quad \hat{y} H_{yn} \tag{3b}$$

where: $$k_c^2 = \frac{m^2\pi^2}{a^2} + \frac{n^2\pi^2}{b^2}$$

a is the larger dimension of the guide
b, guide dimension

The higher modes exist to satisfy the boundary conditions, represent the electromagnetic field disturbances in the neighborhood of a discontinuity, so, the fields in a waveguide can be represented by an infinite series of higher mode waveguide transmission fields, as showed in the following expressions [2]:

$$\begin{aligned}\vec{E} &= \sum_n \vec{E}_{tn}(A_n e^{-j\beta_{mnz}} + B_n e^{j\beta_{mnz}}) \\ &+ \hat{z} \sum_n E_{zn}(A_n e^{-j\beta_{mnz}} - B_n e^{-j\beta_{mnz}})\end{aligned} \tag{4}$$

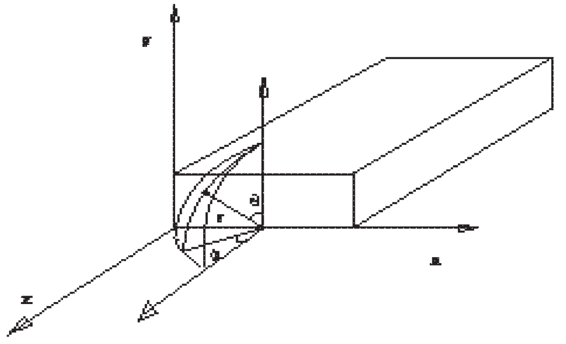


Fig. 2. Coordinate system used in radiation from open ended rectangular waveguide. The radiated electric field will depend of the elevation angle, θ, the azimuth angle, φ, and the distance to the point, *r*.

$$\begin{aligned}\vec{H} &= \sum_n \vec{H}_{tn}(A_n e^{-j\beta z} - B_n e^{j\beta z}) \\ &+ \hat{z} \sum_n H_{zn}(A_n e^{-j\beta z} + B_n e^{-j\beta z})\end{aligned} \tag{5}$$

where: $$\beta_{mn} = \sqrt{\omega^2\mu\varepsilon - \frac{m^2\pi^2}{a^2} - \frac{n^2\pi^2}{b^2}}$$

The equations (4) and (5) are an arbitrary linear sum of a complete set of orthogonal functions. The propagation constant β_{mn} of all terms except that of the dominant mode in the field expansions are imaginary, corresponding to attenuation of the higher mode fields, in such a way, only the dominant mode term will be observable at small distances from the discontinuity, the near field. Therefore, outside the waveguide the higher mode fields can be taken no notice, and the electric field radiated can be treated as an aperture problem.

Working on it, we obtain that the radiated electric field for propagating TE_{10} mode is given by the following expressions:

$$E_\theta = -\frac{\eta\pi a^2 b}{2\lambda^2 r} \sin\phi \left[1 + \frac{\beta_{10}}{k}\cos\theta + \Gamma\left(1 - \frac{\beta_{10}}{k}\cos\theta\right)\right]$$

$$\left[\frac{\cos\left(\frac{\pi a}{\lambda}\sin\theta\cos\phi\right)}{\left(\frac{\pi a}{\lambda}\sin\theta\cos\phi\right)^2 - \left(\frac{\pi}{2}\right)^2}\right]\left[\frac{\sin\left(\frac{\pi b}{\lambda}\sin\theta\cos\phi\right)}{\left(\frac{\pi b}{\lambda}\sin\theta\cos\phi\right)}\right]$$

$$e^{-j[kr-\pi/\lambda \sin\theta\,(a\cos\phi+b\sin\phi)]} \tag{6}$$

And for the radiated electric field, ϕ component:

$$E_\phi = -\frac{\eta\pi a^2 b}{2\lambda^2 r}\cos\phi\left[\cos\theta+\frac{\beta_{10}}{k}+\Gamma\left(\cos\theta-\frac{\beta_{10}}{k}\right)\right]$$

$$\left[\frac{\cos\left(\frac{\pi a}{\lambda}\sin\theta\cos\phi\right)}{\left(\frac{\pi a}{\lambda}\sin\theta\cos\phi\right)^2-\left(\frac{\pi}{2}\right)^2}\right]\left[\frac{\sin\left(\frac{\pi b}{\lambda}\sin\theta\cos\phi\right)}{\left(\frac{\pi b}{\lambda}\sin\theta\cos\phi\right)}\right]$$

$$e^{-j[kr-\pi/\lambda \sin\theta\,(a\cos\phi+b\sin\phi)]} \tag{7}$$

From which, the radiated electric field in the *E* plane ($\phi=\pi/2$) is given by:

$$E_\theta = 2\eta\frac{a^2 b}{\pi^2\lambda r}\sin\theta\left[1+\frac{\beta_{10}}{k}\cos\theta+\Gamma\left(1-\frac{\beta_{10}}{k}\cos\theta\right)\right]$$

$$\left[\frac{\sin\left(\frac{\pi b}{\lambda}\sin\theta\right)}{\left(\frac{\pi b}{\lambda}\sin\theta\right)}\right]e^{-jkr} \tag{8}$$

And the radiated electric field in the plane *H* ($\phi=0$) is:

$$E_\phi = -\eta\frac{\pi a^2 b}{2\lambda^2 r}\left[\cos\theta+\frac{\beta_{10}}{k}+\Gamma\left(1-\frac{\beta_{10}}{k}\right)\right]$$

$$\left[\frac{\cos\left(\frac{\pi a}{\lambda}\sin\theta\right)}{\left(\frac{\pi a}{\lambda}\sin\theta\right)^2-\left(\frac{\pi}{2}\right)^2}\right]e^{-jkr} \tag{9}$$

Notice the term of the second square parenthesis in the plane *E* expression (8), the pattern is determined by the dimension *b* of the guide, and in the plane *H* expression (9), the pattern is determined by the dimension *a*.

B. The reflection coefficient

The reflection coefficient is caused by the signal falls into the guide walls, and the value of it, has been reported by [3]:

$$|\Gamma| = \frac{-c_1+\sqrt{c_1^{\,2}-c_2c_3}}{c_2} \tag{10}$$

where:

$$c_1 = 2\left[C_0+\left(\frac{2}{\pi}\right)^2\left(1+\frac{\beta}{k}\right)\right]\left[\left(\frac{2}{\pi}\right)^2\left(1-\frac{\beta}{k}\right)\right] \tag{11}$$

$$c_2 = \frac{32}{\pi^4}\left[\left(1-\frac{\beta}{k}\right)^2+4\frac{\beta}{k}\left[1+\frac{1}{\sqrt{2}}\left(\frac{k}{\beta}-1\right)\right]^2\right] \tag{12}$$

$$c_3 = 2\left[\left[C_0+\left(\frac{2}{\pi}\right)^2\left(1+\frac{\beta}{k}\right)\right]^2-\frac{64\beta/k}{\pi^4}\left[1+\frac{1}{\sqrt{2}}\left(\frac{k}{\beta}-1\right)\right]^2\right] \tag{13}$$

$$C_0 = \frac{3}{4\pi}\left(\sqrt{\frac{\lambda}{2a}}\right)\left(\frac{\sin(\theta_0/2)}{\cos\theta_0}\right) \tag{14}$$

where:

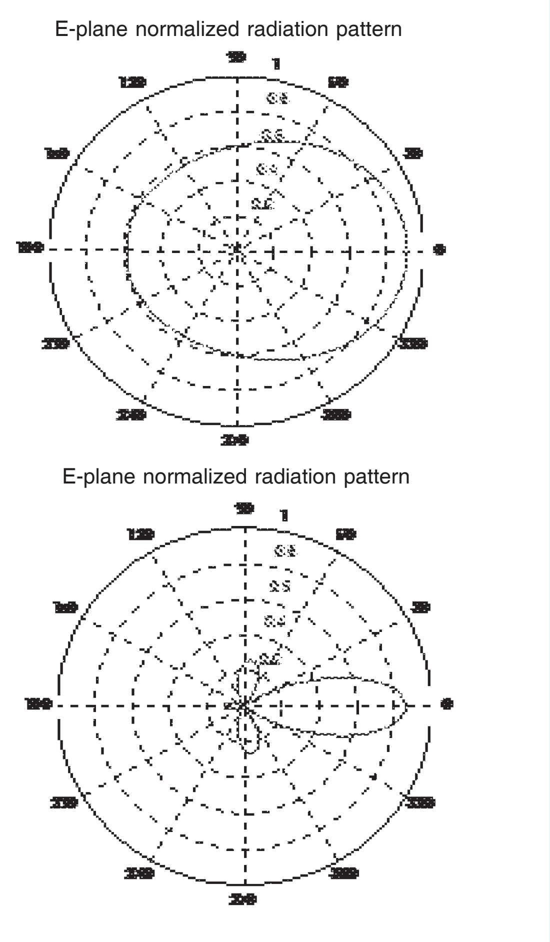
λ ,	free space wavelength
$\theta_0=\cos^{-1}(\beta/k)$,	angle of incidence
$\beta/k=\sqrt{1-(\pi/ka)^2}$	,normalized propagation constant for the TE_{10} mode,
$k=2\pi/\lambda$,	wave number.

C. The radiation pattern

For a rectangular waveguide in *X*-band, the waveguide dimensions are a = 2.286 cm, b = 1.016 cm, operating at a frequency of 9.32 GHz, at a distance of 10 cm, the electric field radiation patterns, propagating TE_{10} mode, *E* plane and *H* plane are shown in figure 3.

5. Aplication

The proposed formulas are useful for the near field antenna measurements, and they can be applied to the indoor environment wireless channel. When the signal propagates through an indoor environment, it consists of multiple replicas of the original transmitted signal, this phenomenon is known


E-plane normalized radiation pattern

E-plane normalized radiation pattern

Fig. 3. Electric field radiation pattern, *E* plane and *H* plane, respectively.

as multipath propagation, and it can cause fluctuations of amplitude, angle of arrival or phase in the received signal.

Then, the indoor environment wireless channel can be understood, such as a hallway, analyzing the electric field radiation pattern. The hallway can be considered as a waveguide, so that, these expressions are an alternative to analyze the indoor environment wireless channel performance.

Applying the worked expressions for an indoor environment, a hallway, a= 3 m, b= 4 with walls made of brick, whose electric permitivity is 4.44 [6], operating at a frequency of 2.4 GHz, and at a distance of 10 m, the electric field radiation pattern, *E* plane is shown in Fig. 4.

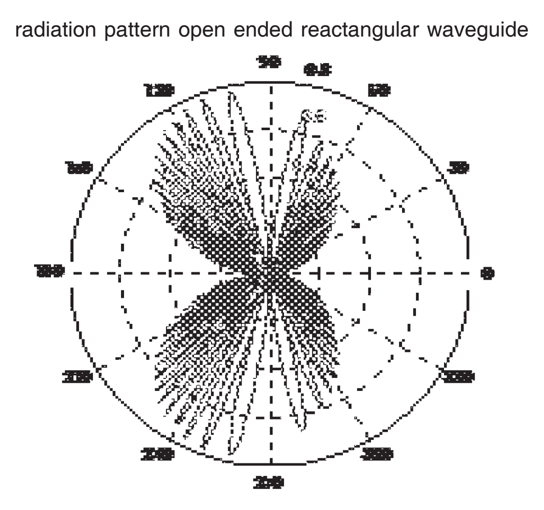

radiation pattern open ended reactangular waveguide

Fig. 4. Radiation pattern when the indoor environment channel is a hallway with walls made of brick.

Making a comparison with the radiation pattern of the electric field radiated from an open ended waveguide, Fig. 3, it shows few side lobes, on the other hand, applying the proposed expressions, the indoor environment radiation pattern shows the multipath effects in the electric field radiated, several nules, each nule is very close to the other, result of the multipaths. The primary lobe is the dominant path and the multipaths are represented by the secondary lobes.

6. Conclusion

Formulas for the electric field radiated from a large open ended rectangular waveguide have been proposed and applied to an indoor environment wireless channel. The incorporation of the reflection coefficient based in the incidence angle, operation frequency and the guide dimension, is the main result of this paper. Numerical examples are given to illustrate the shape of the radiation pattern.

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